Analysis of the application of the Black-Scholes model on the FTSE/ASE-20 stock options of the Athens Derivatives Exchange.

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Abstract
Different models of pricing stock options are tested systematically. The most useful is Black- Scholes model. Using this model to price the behavior of stock options, it is found that pricing errors and implied volatility estimates differ across exercise price and time to maturity. FTSE/ASE-20 were first introduced in August 2000 and nowadays they represent the 91,3% of the Greek options' market. This paper examines the pricing of FTSE/ASE-20 stock options using the Black- Scholes model. It is found that observed prices and predicted prices by the model differ systematically because the model assumes the market to be frictionless. The model overprices both call and put options. Furthermore, implied volatility reduces as contracts become out-of-the-money.

Keywords: Options Market, Stock Options, Black- Scholes Model, Implied Volatility, FTSE/ASE-20, Greece.

Introduction

Over the last 30 years, there has been enormous growth on the use of derivatives instruments and in particular, the use of options contracts. These contracts appear to be valuable for investment management (Neuberger and Hodges, 2002). Investing on options hedge the positions taken in the underlying asset but also make profits in the options market when such opportunities arise (Dritsakis and Grose, 2003). Moreover, they are ideal for hedging future volatility while they are also efficient for gaining lower transaction costs. The ideal strategy is applied according to the future volatility of the security underlying the option. Higher (lower) future volatility guides to higher (lower) option price as well. A successful investor must follow a buying (selling) strategy respectively.

Black was the first author to deal with the valuation of options. A newer and more supplemented model was the Black- Scholes option pricing model. This model assumes that the stock price follows geometric Brownian motion with a constant volatility and the market is frictionless (no transaction costs or differential taxes) (Henderson, 2004). Most of the practitioners use this model because of its simplicity, but numerous studies have proved BS model presents several biases across moneyness and maturity (Xu, 2005). A BS version that eliminates the biases, is the appliance of the implied volatility (variance of the strike price).

The purpose of this paper is to examine the pricing behavior of FTSE-20 options using the BS model. The research areas are the prediction errors, (the difference between the expected price and the observed price), the classification according to their moneyness (the difference between the underlying asset's price and the exercise price) and their implied volatility. In order to highlight these issues, this paper is divided into the following sections. The first part is the introduction to the topic which is followed by the indication of the corresponding background. Thirdly, the data description is mentioned and fourthly the methodology followed. The empirical results are placed in the fifth section and the final section is the report of the conclusions.

Literature Review

Using the market prices to estimate the implied volatilities, the implied values tend to vary across different option series (Duque and Lopes, 2000). Usually, at-the-money options tend to present lower values than in-the-money or out-of-the-money options. Several researches (Macbeth and Merville, 1979 cited by Duque and Lopes, 2000) proved that comparing market warrant prices to theoretical prices resulting bias on option prices. Additionally, Rubinstein, 1985 cited by Duque and Lopes, 2000 produced totally different results for different periods.

Twite (1996), examined the pricing of futures options on share price index (SPI) in the Sydney Futures Exchange (SFE). He took daily closing prices during the period 17 June 1985 till 30 December 1992. Using the Black- Scholes model he tried to price the options and compare the observed prices with the predicted. He concluded that predicted prices differ from the observed prices and tend to overprice in-the-money, underprice at-the-money options and overprice (underprice) out-of-the money call (put) options. Batten and Ellis (2005), used the Black- Scholes model upon the most important currencies traded in spot against US Dollar: The Japanese Yen, the British Pound and the Swiss Franc during the period November 1983 to April 1994. They found out that at-the-money are mispriced by 23%. The results denote that small deviations from statistical independence in asset returns may cause significant economic returns or costs. Also the results underline the need for investors to study the underlying distribution returns when they use short horizon returns in order to estimate long horizon risk. Whaley (1986), studied the S&P 500 equity futures options prices for the year 1983. He found out that early exercise of American futures options causes a significant impact on pricing, under the condition option is at-the-money. He denotes that in-the-money call options are overpriced while out-of-the-money options are underpriced. Regarding puts options the reverse situation exists, that is in-the-money puts are underpriced and out-of-the-money puts are overpriced. The implied volatility is lower for call options than for put options. Dritsakis and Grose (2003, studied the performance of London's options market using the Black- Scholes model during the period January 1995 to December 1999. They found out that BS model can locate possible mispricing of options contracts and specifically over and valued options. Out-of-the-money contracts were undervalued while in-the-money and at-the-money were overvalued. Finally, implied volatility gave more reliable results than historical volatility.

FTSE/ASE-20 Options Market

FTSE/ASE-20 options were established in August 2000. They present 91,3% of the total Greek options market (www.adex.ase.gr). Options are standardized contracts with cash settlement. Their value depends on the value of the underlying asset (FTSE/ASE-20 stock index). In order to calculate the contract size, it is essential to be multiplied with 5€. Options on FTSE/ASE-20 index mature at the close of trading on the last day of trading on the underlying asset (European style). The expiration months are the three nearest consecutive months and the three nearest months from March, June, September and December. The market is marked-to-market with daily margin payments, where the gain or loss (determined by changes in the option premium) is transferred between investors at the close of each day's trading.

Data Description and Pricing Model

The period under study is from 8 August 2004 till 9 August 2006. The data are limited to this period because the Athens Derivatives Exchange (ADEX) database provides daily closing prices of options only for this period. In testing the pricing of options, it is essential to require synchronous observations of both options' and the underlying asset's prices. The sample includes 4970 call prices and 4527 put prices. In general, the predicted price is much higher than the observed price for all kind of contracts. That means that the model overprices both call and put options.

Options contracts are the most difficult to price because each contract has 11 closing prices for calls and 11 closing prices for puts. The Black-Scholes model made things easier because by using its formula, analysts can easily price options. The methodology used is based on Twite (1996), who tried to price future options in Sydney Futures Exchange. Options with daily style margin payments. It is assumed that market is frictionless (no transaction costs or taxes). The Black- Scholes formula is used, which is given by:

$$c(F, X, t, T) = F(t, T)\, N(d_1) - XN(d_2) \qquad (1)$$

$$p(F, X, t, T) = XN(-d_2) - F(t, T)\, N(-d_1), \qquad (2)$$

where: $d_1 = Ln\, [\, F(t, T) / X] + (\sigma_F^2 / 2)\,(T - t) / \sigma_F \sqrt{(T - t)}$

$d_2 = d_1 - \sigma_F \sqrt{(T - t)}$

$c(F, X, t, T)$ the price at time t for a call option that matures at time T

$p(F, X, t, T)$ the price at time t for a put option that matures at time T

$F(T, t)$ the price at time t of the underlying asset

X the exercise price

σ_F the volatility of the daily rate of return on the underlying asset, which is assumed constant over the life of the option contract.

For each option contract the predicted price is estimated from equation 1 for call options and from equation 2 for put options, using the historic volatility to estimate the standard deviation. The observed option price is compared with the predicted price for each day. The difference between the observed option price and the predicted price produces the prediction error. Calls and puts are grouped according to whether they are in-the-money, at-the-money or out-of-the-money. Then follows the classification according to their moneyness(the difference between the underlying asset's price and the exercise price). The last step is their classification according to their moneyness and their implied volatility.

Empirical results

Prediction Errors

Table 1 presents the mean prediction error to be -21,143 for call options while the mean prediction error is -13,8712 for put options. The results infer that across all call and put options, the predicted price is much higher than the observed option price. This means that the model overprices both call and put options. However, it does not mean that FTSE-20 options are going to follow the instruction of the model, as this model is assumed to be frictionless.

Table 1: Pricing of FTSE-20 Options
Prediction Errors

	Number of Observations	Prediction Error (€)				
		Mean	Median	Maximum	Minimum	Range
Call Options						
All	4969	-21,143	-13,13	1707,94	-1716,3	3424,22
F>X	2198	-400,969	-391,509	99,151	-823,961	923,112
F<X	2771	-295,307	-316,616	113,531	-586,511	700,042
Put Options						
All	4527	-13,8712	-19,26	1239,88	-660,31	1900,19
F>X	1829	-274,598	-303,765	1282,734	-792,111	2074,845
F<X	2698	-460,330	-460,930	234,319	-942,677	1176,997

The structure of prediction error is examined also by classifying the euro prediction error according to their moneyness (the difference between the FTSE-20 price and the exercise price) and to days to maturity. Moneyness is given by:

F (t, T) – X / X.

In table 2 the results show that the model tends to underprice in-the-money contracts while it overprices out-of-the-money contracts call options. Based on these results, the rank correlation between predition error and moneyness is -0,091 and -0,312 for in-the-money and out-of-the-money contracts respectively. According to the time horizon, the model tends to overprice long term contracts and either overprice or underprice short term contracts. For put options the model tends to overprice in-the-money contacts and underprices out-of-the-money contracts. The rank correlation between the prediction error and moneyness is -0,697 and -0,244 for in-the-money and out-of-the-money contracts respectively. Regarding the period of time, the model underprices all the contracts except from the middle term contracts (6-10 days).

Table 2: Pricing of FTSE-20 Options
€ Prediction Errors

Moneyness (%)		Days to Maturity							
Low	Up	All	<5	6-10	11-15	16-20	21-25	26-30	>30
Call Options									
31	∞	-618,29	81,025	-15,3609	-93,862	-142,777	-172,312	-265,736	-199,064
26	30	-307,53	18,193	--------	-182,464	-205,571	22,33333	-------	-299,656
21	25	-269,25	1,714	-136,05	-188,738	-252,008	---------	-------	-294,462
16	20	-230,91	-21,806	-139,374	-294,266	-235,985	-314,21	-363,087	-------
11	15	-180,73	-57,283	-237,811	-282,286	-302,116	-418,207	-367,217	-------
6	10	-123,71	-95,280	-279,965	-346,62	-415,904	-486,392	-457,921	-578,447
1	5	-44,257	-139,60	-312,141	-391,032	-444,712	-511,657	-553,819	-587,068

-5	0	44,6407	-155,22	-313,82	-389,628	-441,376	-514,568	-562,81	-590,416
-10	-6	133,369	-101,17	-268,204	-349,125	-410,216	23,10968	-567,511	-563,362
-15	-11	230,393	-104,18	-------	-348,798	-351,566	-410,053	-571,548	-559,627
-20	-16	298,464	11,706	-160,576	--------	--------	-403,625	-------	-------
-30	-21	491,81	-11,92	-158,257	-189,058	--------	-288,35	-------	-------
∞	-31	1477,18	-------	-------	-202,26	--------	55,15366	-------	-------
Put Options									
31	∞	615,832	550,224	-647,658	--------	--------	---------	--------	884,398
26	30	-------	-------	--------	--------	--------	---------	--------	--------
21	25	233,02	-------	--------	368,866	462,996	533,363	--------	--------
16	20	287,515	311,683	-336,284	--------	696,767	867,808	--------	--------
11	15	200,035	301,955	-419,357	508,404	596,16	666,035	652,489	785,963
6	10	127,678	243,592	-404,107	485,892	543,512	611,85	640,102	678,253
1	5	46,0636	186,026	-351,873	426,856	478,937	549,844	601,307	622,065
-5	0	-38,873	103,087	-269,378	340,711	389,194	459,646	511,427	522,756
-10	-6	-129,56	-38,257	-129,476	212,497	257,444	310,623	380,879	405,38
-15	-11	-252,38	-180,57	68,16693	39,9456	--------	183,114	268,733	--------
-20	-16	-348,55	-335,86	135,6099	-117,49	--------	35,2389	--------	175,597
-30	-21	-524,05	-557,44	353,0824	-41,526	-123,224	-266,526	--------	--------
∞	-31	-870,48	-856,64	1060,639	-672,476	-765,455	-646,029	--------	--------

Figure 3,4 plots the prediction error for call and put options by the degree of moneyness. It is obvious that mispricing exists in all levels of moneyness but the most usual interval is between -5% till 35% for calls and -20% till 15% for puts. As far as time is concerned, figure 5 mentions that mispricing is independent of time horizon, as the prediction error fluctuates between -450 till 250, even when the contract is stature or not.

Figure 1: Call Options Prediction Error in Relation to Moneyness

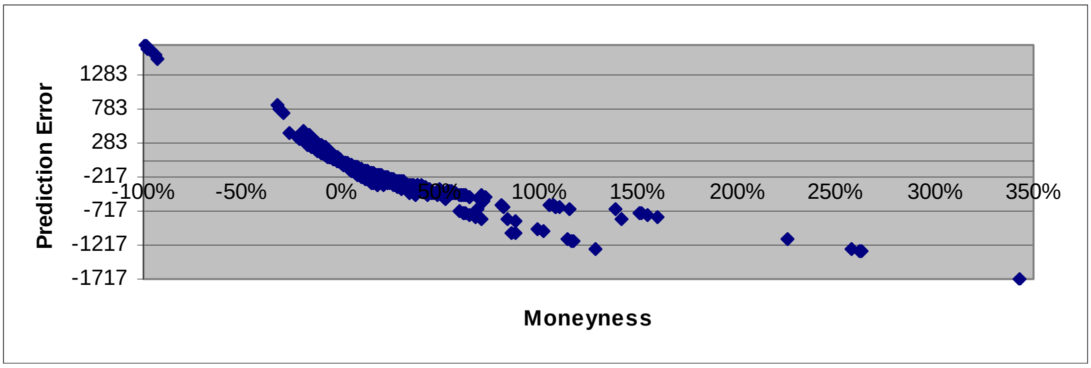


Figure 2: Put Options Prediction Error in Relation to Moneyness

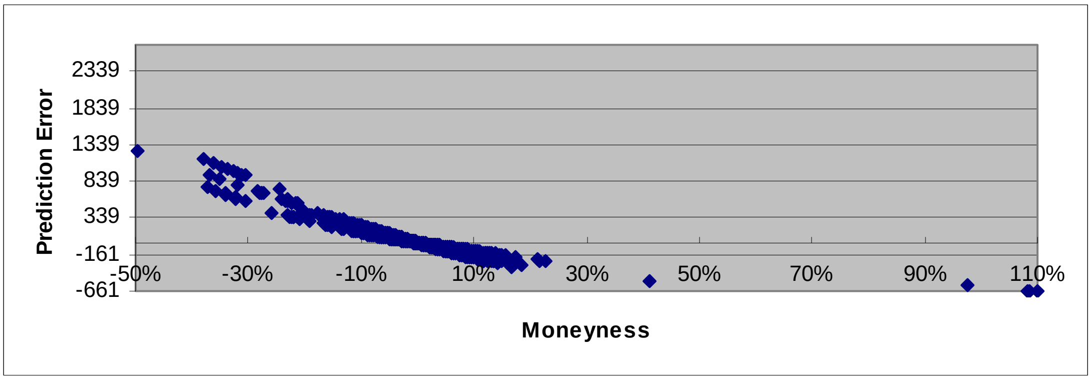


Figure 3: Call Options Prediction Error in Relation to Days to Maturity

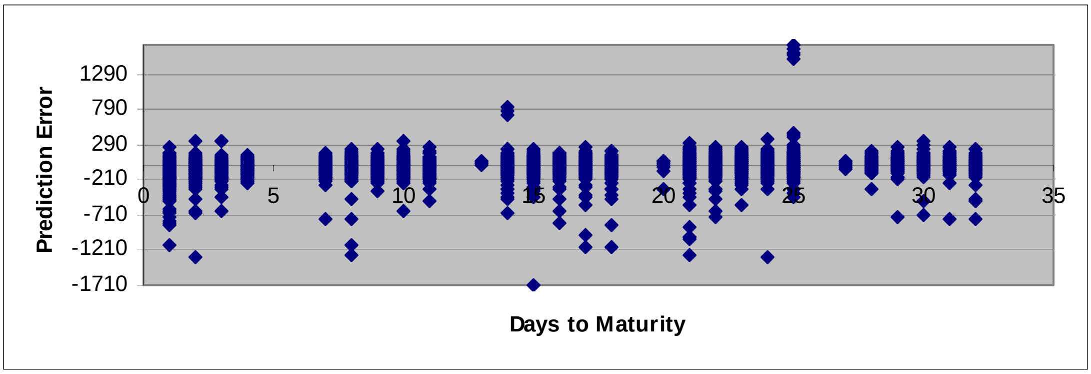
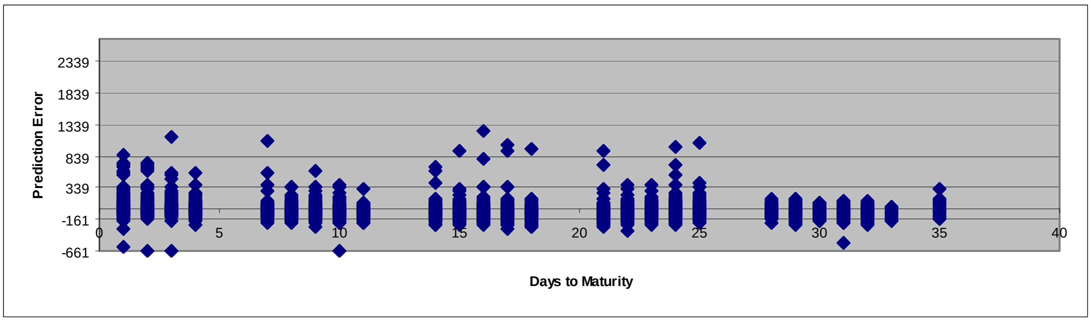


Figure 4: Put Options Prediction Error in Relation to Days to Maturity



Implied Volatility

Implied volatility is different from historic volatility and it changes every day. It does not track a direction but it reflects the price of the option rather than the price of the underlying asset. In general it increases when the market moves downward and decreases when the market moves upward. It is given by:

$x_{i+1} = x_i - (y_i - p) / v_i$

where: p the option's price

x_i the volatility

y_i the option's theoretical value at volatility x_i

v_i the option's vega at volatility x_i.

The implied volatility in this case is calculated based on equations 1,2 and in table 3 the implied volatility and its relation with moneyness and time to maturity are summarized. In general, implied volatility is high for in-the-money call options and it reduces continuously as the contracts become out-of-the-money. Regarding puts, implied volatility is high for in-the-money contracts, becomes null for at-the-money contracts, gets negative and continuously increases for out-of-the-money contracts. Call options behave the same, either they are short or long and in-the-money present high implied volatility while out-of-the-money negative. Put options present high implied volatility without exception of time period. The rank correlation between implied volatility and moneyness for calls is 0,120 and 0,722 for in-the-money and out-of-the-money respectively, while the corresponding numbers for puts are 0,703 and 0,716. All the above results are contrary with the results of Twite (1996) in Australian Stock Exchange. Specifically, Twite proved that the model undeprices options contracts and generally overprices in-the-money contracts and underprices out-of-the-money contracts.

Table 3: Implied Volatility for FTSE-20 Options

Moneuness (%)		Days to Maturity							
L	U	All	<5	6-10	11-15	16-20	21-25	26-30	>30
					Call Options				
31	∞	50,89%	108,46%	32,07%	32,25%	28,48%	26,95%	32,07%	23,04%
26	30	43,07%	50,28%	-------	-------	24,71%	27,68%	------	32,23%
21	25	41,01%	50,87%	33,06%	29,05%	17,95%	-------	------	32,00%
16	20	43,15%	54,01%	16,20%	21,11%	34,59%	19,06%	28,62%	-------
11	15	27,63%	13,17%	22,12%	21,60%	16,82%	17,68%	------	-------
6	10	17,75%	8,01%	17,69%	14,36%	13,58%	13,70%	13,15%	7,32%
1	5	12,32%	2,69%	12,07%	11,58%	11,89%	11,83%	12,21%	2,62%
-5	0	9,18%	-2,09%	-2,43%	9,20%	2,61%	9,56%	9,2%	-2,89%
-10	-6	7,66%	-7,37%	-7,63%	7,76%	7,86%	8,26%	8,34%	-8,63%
-15	-11	7,67%	10,15%	-------	7,42%	5,41%	7,73%	7,60%	-11,79%
-20	-16	6,63%	19,80%	0,51%	-------	-------	5,52%	------	-------
-30	-21	1,54%	-2,51%	7,43%	1,41%	-------	5,40%	------	-------
∞	-31	-0,69%	-------	-------	-13,86%	-------	1,95%	------	-------
					Put Options				
31	∞	21,94%	76,94%	42,18%	-------	-------	-------	------	32,86%
26	30	-------	-------	-------	-------	-------	-------	------	-------
21	25	21,83%	-------	-------	37,96%	36,17%	34,47%	------	-------
16	20	9,05%	52,94%	38,29%	-------	35,36%	34,41%	------	-------
11	15	6,16%	41,94%	37,48%	35,95%	35,03%	34,13%	33,10%	32,75%
6	10	4,71%	40,25%	36,31%	35,13%	34,57%	33,69%	32,96%	32,65%
1	5	2,65%	36,50%	34,99%	34,33%	33,92%	33,28%	32,72%	32,48%
-5	0	-0,20%	32,83%	33,53%	33,35%	33,15%	32,77%	32,38%	32,21%
-10	-6	-3,80%	27,88%	31,78%	32,22%	32,32%	32,15%	32,02%	31,87%
-15	-11	-12,14%	23,41%	29,72%	30,82%	-------	31,45%	31,68%	-------
-20	-16	-15,16%	18,45%	28,54%	29,23%	-------	30,585	------	31,9%
-30	-21	-25,34%	12,56%	26,49%	27,61%	29,22%	29,11%	------	-------
∞	-31	-48,43%	1,61%	19,82%	26,50%	25,47%	27,97%	------	-------

Conclusion

This paper tried to evaluate the Greek options' market using the most common used pricing model, the Black-Scholes model. This evaluation proved that the observed prices differ from the predicted prices. Specifically, the model overprices both call and put options contrary to the results presented by Twite, 1996 and Whaley, 1986. Specifically, out-of-the-money call options are overpriced while in-the-money call options are underpriced. Put options present the altogether different results. The prediction error according to the moneyness and the days to maturity, show that short term call options are either underpriced or overpriced while long term contracts are overpriced. Put options are underpriced except from the middle term contracts (6-10 days). Also the correlation between the moneyness and the prediction error proved to be negatively weak for call options and negatively strong for put options. Mispricing exists in all levels of time maturity, but the most usual intervals of mispricing exist between -5% till 35% and -20% till 15% for call options and put options respectively. Implied volatility is high for in-the-money contracts and reduces as contracts become negative. This paper proved that Black- Scholes model produce totally different results than the observed prices in the market.

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