# Methods of Avoiding Insolvability in Life Insurance

**Dragos Simona**
Department of Finance
Babes Bolyai University, Faculty of Economics, Cluj Napoca, Romania
Laboratory of Economics Orleans (LEO)
Faculty of Economics, Administration and Law, University of Orleans, France
sfarcas@econ.ubbcluj.ro

***Abstract***
*In this article we will demonstrate the utility of an optimal allocation of actives and the utility of reinsurance as ways of avoiding the insolvability. For an optimal allocation of actives we will model the behaviour of the insurer by maximising the expected return of own funds and by minimising the insolvability risk. In order to explain the necessity of reinsurance in life insurance we will explain the link between the security coefficient of a company and its ruin probability.*

Keywords: insolvability, life insurance, reinsurance, allocation assets, risk

## 1. Introduction

The risk is the essential and characteristic element of an insurance contract. Being a future and uncertain event, independent of the parts' will, it constitutes the foundation of the insurance contract[1]. The central concept of the risk theory is the "ruin probability". An important part of the works on the risk theory tries to minimise the ruin probability of an insurance company. Even if the classical risk theory was reduced to a reinsurance theory - Borch (1962), the dynamic and modern approach offers other solutions in order to minimise the probability that a company goes bankrupt (insolvable).

Like any other company, a life insurance company is insolvable when the debts exceed the accounts receivable. The extent of insolvability is the difference between these two quantities. The existence of the insolvability risk justifies the prudential legislation in insurances, taking the shape of the solvency margin, which imposes to the insurance company the detention of a minimal value of own funds as a function of the engagements to the insurers.

The solvability of an insurance company may be affected by many factors. If we step aside the wastages due to the operational risks we can distinguish two types of essential losses[2]:

1 Losses due to the set of positions which the society holds in different market instruments (actions, bonds, derivatives). This portfolio can suffer losses due to the market risks, namely a disadvantageous evolution of the company assets. The insurance

[1] Brière de l'Isle, G., (1973), *Droits des Assurances*, Presses Universitaires de France.
[2] Ory, J.N., (2002), ‚La démarche RAROC utilisée en banque est-elle transposable à l'assurance vie?', Research document, nr.10, GREFIGE-Nancy 2 University.

company needs to ***evaluate its need of own funds*** for its investments for an ***optimal allocation of its assets.***

2 Losses due to an underestimation of its engagements or an improper evaluation of the behaviour of the insured. In this case we talk about a passive risk, translated in unexpected losses for the company assets, especially for the titles which seemed risk-free because their expiration and period matched those of the scheduled engagements. In order to avoid these situations we have to constitute own funds. The increase of own funds appealing to the stockholders is a limited resource. In the short term and for known own funds, the ***reinsurance*** is the most adequate modality to action directly to the structure of risks without modifying the portfolio.

## 2. Optimal allocation of assets in life insurances

In order to avoid insolvability, the insurer needs to allocate its assets in order to minimise the loss risk. For an optimal allocation of the assets the behaviour of the insurer can be modelled by the following optimisation program[3]:

1 Maximisation of the expected value of the efficaciousness of own funds;

2 Minimisation of the insolvability risk.

The optimisation program can be expressed as follows:

1 Max $E(r_{fd})$ where $r_{fd}$ represents the efficaciousness of the own funds deferring to the second condition;

2 $P(r_{fd} < r_c) < k$ .

The constraint « safety first », chosen for administrating the bankruptcy risk corresponds to a ruin probability which depends on the values of $r_c$ and $k$ - the limit of risk accepted by the control organs, where $r_c$ - the minimal rate of efficaciousness that the insurer can accept.

The preceding constraint implies using the Bienaymé-Cebîsev inequality that:

$$E(r_{fd}) - \frac{\sigma_{fd}}{\sqrt{k}} \geq r_c \ .$$

The balance sheet of a life insurance company can be written as follows:

| **Assets** | **Liabilities** |
|---|---|
| **A** – Assets | **F** – Own funds<br>**L** –Liabilities from the insureds |

If we consider an annual model, the balance sheet at the beginning of the year can be written:

[3] Albizzati, M.O., (1996), 'Quelques aspects de la gestion des risques en assurance: une approche financière et probabiliste', Ph.D. Thesis sustained at the Reims University.

$A = F + L$ .

For each year we will consider the new premiums (subscriptions) - $P$ - and the total of outlets - $I$ - including the finished contracts and the anticipated ransoms.

If $P < I$ , that is in case of massive ransom, the outlets will be financed by selling the assets at the market value. The premiums and the falling due contracts are registered at the end of the year.

The equation which describes the company income is:

$r_{fd} \cdot F = r_a \cdot A - r_u \cdot L$ , where $r_a -$ the efficaciousness of assets,

$r_u -$ the remuneration rate of the insureds.

The activity of the insurance company can be expressed by the following equation:

$$A(1 + r_a) + P = (L - I)(1 + r_u) + P + I(1 + r_u) + F(1 + r_{fd}).$$

The asset A grows with the new written premiums. The mathematical reserves from the liabilities grow as new premiums are written, the falling due contracts $S$ and the contracts from the portfolio $(L - I)$ are remunerated at the same rate $r_u$ (if there aren't ransom penalties), the own funds being valorised at the rate $r_{fd}$ .

At the beginning of the next year, after taking into consideration the premiums and the outlets, the balance sheet is:

$$A(1 + r_a) + P - I(1 + r_u) = (L - I)(1 + r_u) + P + F(1 + r_{fd}).$$

If $P > I(1 + r_u)$, the asset can be conserved in portfolio. Otherwise a part of the asset has to be given up in order to finance the outlets. If these outlets are correctly anticipated, they can be covered by a bonded asset of the same maturity. For the part $I(1 + r_u)$ of the unanticipated and uncovered outlets the insurer must sell the corresponding asset at his market value which can be depreciated. We suppose that the outlets have been correctly anticipated.

An important element of the optimal assets allocation process is constituted by the determination of the proportion of stocks (shares) in the portfolio.

The stocks will have the weight $a$ in the portfolio with the efficaciousness of the market portfolio - $r_m$, and the bonds will be modelled by an asset of efficaciousness equal with the risk-free interest rate $r_f$ , if there aren't anticipated ransoms - $E(r_m) > r_f$ :

$$r_a = a \cdot r_m + (1 - a) \cdot r_f \ .$$

To obtain a better approximation of the risk from the bonds we have to take into consideration the market value of the titles before the maturity in case of ransom.

By combining a placement in the risk-free asset (rentability $r_f$ ) and a stock portfolio (expected rentability $E(r_m)$ , risk $\sigma_m$ ), the insurance company anticipated a rentability $E(r_a)$ for the set of placements, so that:

$$E(r_a) = a \cdot E(r_m) + (1 - a) \cdot r_f \ .$$

We will express the efficaciousness of the own funds as a function of the remuneration rate of the insureds and the assets efficaciousness:

$$r_{fd} = (1+\frac{L}{F})\cdot r_a - \frac{L}{F}\cdot r_u.$$

The remuneration rate of the insureds is modelled by $r_u = Max(r_{\min}, r_a),$ where $r_{\min}$ is the guaranteed interest rate by the insurer. The efficaciousness of the owned fund is:

$$r_{fd} = (1+\frac{L}{F})\cdot r_a - Max(r_{\min}, r_a)\cdot\frac{L}{F}.$$

There are two cases: the efficaciousness of assets is superior to the minimal guaranteed rate.

- If $r_a < r_{\min}$, $r_{fd} = r_a - l(r_{\min} - r_a)$, where $l = \frac{L}{F}$ is the company's leverage factor. In this case $\beta_e = (1+l)\cdot\beta_a = a\cdot(1+l)$ and $\sigma_{fd} = (1+l)\cdot\sigma_a$.

The expected value of the efficaciousness of the owned funds is expressed in function of $a$:

$$E(r_{fd}) = E(r_a) - E[l(r_{\min} - r_a)].$$
$$E(r_{fd}) = a\cdot(1+l)\cdot\lfloor E(r_m) - r_f\rfloor + r_f\cdot(1+l) - l\cdot r_{\min}.$$

If we take into consideration the second constraint the proportion of stocks in the portfolio is:

$$a \le \frac{(1+l)\cdot r_f - l\cdot r_{\min} - r_c}{(1+l)\cdot\left[r_f + \frac{\sigma_m}{\sqrt{\theta}} - E(r_m)\right]}.$$

- If $r_a \ge r_{\min}$, that is if the efficaciousness of the asset is superior to the minimal guaranteed rate, the efficaciousness of the own funds is $r_{fd} = r_a$. In this case, $\beta_{fd} = \beta_a$ and $\sigma_{fd} = a\cdot\sigma_m$.

The control constraint of the risks is less powerful and is reduced to:

$$a \le \frac{r_f - r_c}{\left[r_f + \frac{\sigma_m}{\sqrt{\theta}} - E(r_m)\right]}.$$

The solution $a$ gives the optimal proportion of stocks that the insurer must include in its asset portfolio as a function of its owned funds.

A central part of the risks administration implies the internal evaluation of the financial resources in order to avoid the difficulties and the financial insolvability.

## 3. The utility of reinsurance in life insurances

A second method of avoiding the insolvability risk is the reinsurance. In order to explain the necessity of reinsurance in life insurances we will explain the link between the security coefficient of an insurance company and its ruin probability.

The level of the insurers expenses – ($X$) – can exceed the level of the premiums from a certain period of time because the premiums are fixed at subscription and the expenses can fluctuate. Because the insurer can suffer losses he must dispose of own funds in order to cope with them. The own funds are composed of own capitals subscribed by the shareholders and of the reserves which represent benefits obtained in the past and not distributed. The ruin appears if the annual loss exceeds the level of owned funds (*F*). Such an event has the following probability:

$$P(R < -F) = P(R < -\frac{F + E(R)}{\sigma(R)}),$$ where R - the result of the insurer.

The result of the insurer is a random variable:

$$R = n_a\pi + n_a\lambda\pi - \sum X_i ,$$ where $n_a$ - number of insured's

$\pi = E(X)$ - pure premium

$\lambda$ - the pricing coefficient

In what follows we will consider the example of a temporary life insurance, for which we will establish the individual risk and the total risk for the set of contracts of the same type.

The insurer guarantees $n_a$ identical and independent risks: each insured *i* $(i = 1,....,n_a)$ has a probability *q* to die during the year. In this case, the insurer will pay the sum *c* to the beneficiary of the contract. The cost *c* for the insurer is not random, while the moment of occurring is random.

**The individual risk**

The random annual expense corresponding to an insured *i*, named $X_i$, can take only two values: *c* with the probability *q* or 0 with the probability *1-q*. $X_i$ follows a Bernoulli law $B(1,q)$, with multiplicative factor *c*. As a consequence we can write:

$E(X_i) = q \cdot c$ and $\sigma(X_i) = c \cdot \sqrt{q(1-q)}$ .

If *q* is small,

$$\sigma(X_i) \approx c\sqrt{q} \quad \text{şi} \quad \frac{\sigma(X_i)}{E(X_i)} \approx \frac{1}{\sqrt{q}} .$$

**The total risk**

The random annual expense corresponding to $n_a$ insureds, named $\sum X_i$, is the increment between the sum *c* and the binomial variable of parameters $n_a$ and *q*. In consequence:

$E(\sum X_i) = n_a qc$ and $\sigma(\sum X_i) = c\sqrt{n_a q(1-q)}$ .

If *q* is small,

$$\sigma(\sum X_i) \approx c\sqrt{n_a q} \quad \text{and} \quad \frac{\sigma(\sum X_i)}{E(\sum X_i)} \approx \frac{1}{\sqrt{n_a q}} .$$

In practice a binomial law can be considered a normal law if $n_a q > 3$, condition verified if for example $n_a > 1000$ and $q \approx 0{,}01.$

In order to illustrate the link between the solvability and reinsurance we will define the security coefficient of the company starting from the ruin probability.

The ruin probability can be expressed as $P(R < -\frac{F+E(R)}{\sigma(R)})$. The security coefficient[4] is:

$\beta = \frac{F+E(R)}{\sigma(R)}$, where *F* – own funds of the insurer

*R*- the result of the insurer.

For the case of the temporary life insurance the security coefficient for the total risk is $\beta = \frac{F + n_a cq\lambda}{c\sqrt{n_a q(1-q)}}$. The legislation usually imposes the minimal level of this security coefficient, level upon which the company is in danger to be bankrupt.

So for risks of given nature ($E(X)$ and $\sigma(X)$ known) the insurer must maintain the security coefficient $\beta$ at a satisfactory level[5]:

- either increasing the owned funds *F* appealing to shareholders (limited resource by the will of shareholders );
- either increasing the insurance prime – the pricing **coefficient,** $\lambda$**;**
- or increasing the number of insureds, $n_a$.

For fixed owned funds, the last two solutions are difficult to put into practice. The increasing of the premium deteriorates the competitiveness and leads to the reduction of the number of subscribed contracts (migration of the insureds towards the competition) and to the reduction of the security coefficient. The rapid increasing of the number of contracts, even through non-pricing methods, is risky in insurance. The most elastic part of the demand is in fact constituted by the risks that are hard to estimate. The structure of risks can be unfavourable modified. The only way of adjusting the security coefficient in the short term and for fixed own funds is to action directly on the risks structure without modifying the portfolio by reinsurance.

### Diminishing the ruin probability through reinsurance

Reinsurance is an operation through which the insurer assures himself against the effects of the written insurances. The objective of the reinsurance is the reduction of the fluctuation of the random results of the insurer. The reinsurance can be used for financing the launching of new types of contracts. It allows to lessen the constraints of the solvency margin because the premiums given to the reinsurer won't be taken into consideration for the calculus of this margin. The reinsurance implies a risk transfer and a benefits transfer. An optimal reinsurance strategy means the arbitrariness between two effects: a positive one (risk transfer) and a negative one benefits transfer). We will show the effect of the reinsurance quote-part for the result of the insurer.

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[4] Tosetti, A., Behar, T., Fromenteau, M., Menart, S., (2002), *Assurance -Comptabilité, Réglementation, Actuariat,* Economica, 2nd edition, Paris.

[5] Deelstra, G., Plantin, G., (2006), *Théorie du risque et réassurance*, Economica, Paris.

The quote part reinsurance is the most simple and used form of reinsurance: the insurer cedes to the reinsurer a determined part of each prime; the reinsurer engages to pay the same part from the engagements generated by each event. The ceded part is constant, independent of the insured risk.

We suppose that the insurer cedes a part $(1-\theta)\cdot\sum_{i=1}^{n}E(X_i)$ from the received premiums and keeps the part $\theta\cdot\sum_{i=1}^{n}E(X_i)$. Therefore, the insurer will get $(1-\theta)\cdot\sum_{i=1}^{n}X_i$ from the engagements towards the insureds and will have to cover debts toward insureds that amount to $\theta\cdot\sum_{i=1}^{n}X_i$. $\theta$ is the retention coefficient of the insurer. The result of the reinsurer (R), is a random variable: $R=\sum_{i=1}^{n}E(X_i)-\sum_{i=1}^{n}X_i-C_g$, where $C_g$ - administration expenses.

The insurer cedes a part $(1-\theta)$ from its total premiums $\sum_{i=1}^{n}E(X_i)$ and the same proportion $(1-\theta)$ from its administration expenses $g\cdot\sum_{i=1}^{n}E(X_i)$, which balances the total administration expenses $C_g$. The reinsurer reflows not only a part of the administration expenses $(1-\theta)\cdot g\cdot\sum_{i=1}^{n}E(X_i)$ that receives, but also a part $c_r$, named reinsurance commission, from the ceded primes that he received $(1-\theta)\cdot\sum_{i=1}^{n}E(X_i)$.

**Table 1: The result of the insurer before and after reinsurance**

| | **Before reinsurance** | **After reinsurance** |
|---|---|---|
| Prime | $\sum_{i=1}^{n}E(X_i)$ | $\theta\cdot\sum_{i=1}^{n}E(X_i)$ |
| Engagements | $-\sum_{i+1}^{n}X_i$ | $-\theta\cdot\sum_{i+1}^{n}X_i$ |
| Administration expenses | $-g\cdot\sum_{i=1}^{n}E(X_i)$ | $-g\cdot\sum_{i=1}^{n}E(X_i)+c_r\cdot(1-\theta)\cdot\sum_{i=1}^{n}E(X_i)$ |
| Result | $R=\sum_{i=1}^{n}E(X_i)-\sum_{i=1}^{n}X_i-g\cdot\sum_{i=1}^{n}E(X_i)$ | $R^r=\theta\cdot\sum_{i=1}^{n}E(X_i)-\theta\cdot\sum_{i=1}^{n}X_i$ $-(g-c_r(1-\theta))\cdot\sum_{i=1}^{n}E(X_i)$ |

Source: Tosetti, A., Behar, T., Fromenteau, M., Menart, S., 2002, *Assurance -Comptabilité, Réglementation, Actuariat,* Economica, (2d ed.), Paris.

The interest of the reinsurance is the decreasing of the ruin probability, whose price is generally the diminution of the expected value of the benefit.

If $c_r = g$, then $E(R^r) = \theta \cdot E(R)$ and in all cases $\sigma(R^r) = \theta \cdot \sigma(R)$, so:

- the expected value of the benefit is smaller than before the reinsurance:
  $E(R^r) = \theta \cdot E(R) < E(R);$

- therefore the result is less dispersed and the security coefficient grows:
  $E(R^r) = \theta \cdot E(R) < E(R);$

Replacing in the security coefficient results:

$$\beta^r = \frac{FP + \theta \cdot E(R)}{\theta \cdot \sigma(R)} > \beta = \frac{FP + E(R)}{\sigma(R)}.$$

If $c_r > g$ two assertions are verified:

$$E(R^r) > \theta \cdot E(R)$$

$\sigma(R^r) = \theta \cdot \sigma(R)$, from where results $\beta^r > \beta$.

But if $c_r < g$ we cannot conclude that $\beta^r > \beta$.

In the extreme case in which the insurer cedes 100% of the risks, his probability of ruin is 0 but he cannot expect any benefit.

For quote part reinsurance, there is a maximal retention coefficient if the insurer wants to be fully covered against the ruin risk:

$$\theta_{\max} = \frac{FP}{\beta \cdot \sigma(R) - E(R)}.$$

An advantage of this type of reinsurance is the ease to apply it. In this case the insurer and the reinsurer have exactly the same rate $\frac{damages}{premiums}$. From this property result another two advantages:

- the insurer and the reinsurer have equal interests and the covering by reinsurance doesn't incite the cedant to adopt an unfavourable behaviour against the reinsurer as long as its retention rate is sufficient;
- this identity is not the best way of reducing the volatility of the net portfolio. The other forms of reinsurance break this symmetry letting the most risky part of the insured events to the reinsurer.

*The necessity of the reinsurance commission*

Under this form the reinsurance quote part is not very equitable for the insurer because he has to completely administrate its portfolio, including the ceded part. Its administration and acquisition expenses, which he has to assume alone, are in theory covered by the supplementary expenses associated to the pure premium. For the reinsurer the administration is simple and is inequitable to get all the supplementary expenses over the ceded premiums.

The reinsurance commission eliminates this disadvantage. The reinsurer indemnities the cedant for the administration of its premiums by retrocessing a percent $c_r$ from the ceded premiums, named commission rate.

Thus,

- if the commission rate is equal to the administration expenses of the cedant, $g$, the contract is completely proportional;
- if $c_r < g$, the insurer cedes more benefit than activity;
- if $c_r > g$, the insurer increases its commercial rentability (result/net activity) by reinsurance.

The quote part reinsurance is used mostly for financing a new branch of activity. The ceded proportion and the commission rate strongly decrease over time. The highly ceded proportion allows saving from the immobilisation of the own funds imposed by the prudential legislation.

The very high rate of commission at the beginning of the reinsurance period is explained by a financing done by the reinsurer, who in the first years accepts a negative balance of reinsurance, which in the second part of the period, is compensated by a claims' improvement and by a commission inferior to the administrating expenses.

## 4. Conclusions

In this article we have presented two possibilities for minimising the insolvability risk of a life insurance company:

- *practising an optimal allocation of assets,* which means the minimisation of the insolvability risk and a maximisation of the expected value of the efficaciousness of the owned funds;
- *using reinsurance in life insurances*, whose utility has been explained throughout the security coefficient of an insurance company and its link with the ruin probability. The ruin appears if the annual loss exceeds the level of owned funds. The probability of ruin is *P (insurer result < - security coefficient)*. The minimal level for this coefficient is given by the legislation. The only way of adjusting the security coefficient for a certain level of own funds is the reinsurance.

In conclusion, the reinsurance is not the only way of reducing the insolvability as stipulated by the classical risk theory. An optimal allocation of the assets manages to handle the problem efficiently.

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## Curriculum Vitae

### Personal Information

Name : Dragos (Farcas)Simona Laura
Adress : Department of Finance
Faculty of Economics
Babes-Bolyai University
Street Teodor Mihali, 58-60
400591, Cluj-Napoca, Romania
Tel.: 40+0264 41.86.52/3/4/5
Fax: 40+0264 41.25.70
Email: sfarcas@econ.ubbcluj.ro
Date of birth: 01,09,1979

### Studies

2002- present: Ph.D. Student:-Department of Finance, Babes-Bolyai University, Cluj-Napoca, Romania
-Laboratoire d'Economie Orléans (LEO), University of Orleans, France
2001-2002: Master studies: Financial Strategies in Business, Faculty of Economic, Babes-Bolyai University, Romania
1997-2001: Bachelor in Finance: Banks and Stock Exchanges, Faculty of Economic, Babes Bolyai University, Romania

### Professional Experience

2002- present: Assistant Professor at the Department of Finance, Babes-Bolyai University, Cluj-Napoca, Romania